A Combined Packed-Bed Friction Factor Equation: Extension to Higher Reynolds Number with Wall Effects

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Significance

A unique combination of classic packed bed friction factor equations and newly refit correlation constants is proposed which produces a new friction factor correlation which significantly improves predictions in high turbulence regimes, high porosity regimes, and high wall effect regimes.

Keywords: packed-bed reactor, friction factor, pressure drop, particle technology, fluid mechanics

Published Correlations

Pressure drop in a packed bed is typically estimated by using Eq.1 with an appropriate correlation for the friction factor (f_k)

$$\frac{\Delta P}{L} = \frac{\rho u^2 (1-\varepsilon_b)}{d_{p_e} \varepsilon_b^3} f_k \tag{1}$$

In the past, many friction factor correlations which describe the dependence of f_k on a variety of bed and fluid properties have been proposed for packed-bed pressure drop estimation.[1] The most universally used is the Ergun equation, Eq. 2.[2]

$$f_k = \frac{150(1-\varepsilon_b)}{\mathrm{Re}} + 1.75 \tag{2}$$

Over the years, modifications to this correlation have attempted to correct for deviations between predicted values and experimental data observed under some specific conditions. These include high Re and low tube diameter to particle-diameter ratios (d_t/d_p). Tallmadge[3] (Tal) in 1970 showed that the applicable range of $Re/(1-\varepsilon_b)$ for the Ergun equation was from 0.1 to 500. He extended this range to 0.1–100,000 by modifying the turbulent term of the Ergun equation (1.75) to be a function of Re, resulting in his correlation, Eq. 3

$$f_k = \frac{150(1-\varepsilon_b)}{\mathrm{Re}} + \frac{4.2}{\left(\frac{\mathrm{Re}}{1-\varepsilon_b}\right)^{1/6}} \tag{3}$$

Other improvements have been made to address packed-beds with $d_t/d_p < 10$, where flow interactions with the wall and channeling in addition to interactions with the particles have significant effect on pressure drop. In this article, we have considered three of these correlations, including one from Mehta and Hawley[4] (MH) and two from Liu et al.[5] The two from Liu et al. include the same wall effect terms. One correlation used the same constants and form as the Ergun equation and will be termed the Liu-modified Ergun (LME) correlation, Eq. 4

$$f_k = \frac{150(1-\varepsilon_b)}{\mathrm{Re}} \left(1 + \frac{\pi d_{p_e}}{6 d_t (1-\varepsilon_b)}\right)^2 + 1.75 \left(1 - \frac{\pi^2 d_{p_e}}{24 d_t}\left(1 - 0.5\frac{d_{p_e}}{d_t}\right)\right) \tag{4}$$

The second correlation uses an original form and will be termed the Liu correlation. It is noteworthy that these three correlations only attempted to fit data in the laminar flow regime.

Proposed Combined Correlation

These different modifications give the possibility of two different combinations of the high Re modification with a wall-effect modification: Tal with the MH wall-effect terms (TMH) and Tal with the LME wall-effect terms (TL). The preferred combined correlation as proposed by this work is TL which includes the high Re modification from Tal, Eq. 3, with the low d_t/d_p modification of LME, Eq. 4, into a single correlation, Eq. 5, valid over a wide range of Re and d_t/d_p ratios

$$f_k = \frac{K_{\mathrm{Lam}}(1-\varepsilon_b)}{\mathrm{Re}} \left(1 + \frac{\pi d_{p_e}}{6 d_t (1-\varepsilon_b)}\right)^2 + \frac{K_{\mathrm{Turb}}}{\left(\frac{\mathrm{Re}}{1-\varepsilon_b}\right)^{1/6}} \left(1 - \frac{\pi^2 d_{p_e}}{24 d_t}\left(1 - 0.5\frac{d_{p_e}}{d_t}\right)\right) \tag{5}$$

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Table 1. Ranges of Bed Porosity, d_t/d_p, and Re for the Experimental Data Sets

	Ergun and Orning	Burke and Plummer	Wentz and Thodos
# points	52	175	97
ε_b	0.33–0.35	0.36–0.42	0.35–0.88
d_t/d_p	45–50	8.3–35	11
Re	0.32–29	1.4–970	1500–7700

The TMH correlation is not given and will not be discussed further in this article but data for this correlation are included in Table 2 for comparison. In Eq. 5, the term in red shows the Tal modification and the terms in blue are those added from the LME correlation.

Preferred values of the two constants, K_{Turb} and K_{Lam}, in the TL correlation, Eq. 5, were obtained by fitting published $\Delta P/L$ data and minimizing the overall average relative absolute error (ARAE), Eq. 6

$$\mathrm{ARAE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|x'_i - x_i|}{x_i} \tag{6}$$

The resulting values of the fitted constants ($K_{\mathrm{Lam}} = 119.8$ and $K_{\mathrm{Turb}} = 4.63$) differed slightly from those given by Ergun ($K_{\mathrm{Lam}} = 150$ and $K_{\mathrm{Turb}} = 1.75$) with the K_{Turb} value matching closely the value ($K_{\mathrm{Turb}} = 4.2$) used by Tallmadge (Eq. 3). Below, we compare the performance of this new combined correlation (TL) using our fitted constants to the performances of the other published equations in predicting a large set of pressure drop data from several published sources.

Published Data Sets

Pressure drop data, totaling 476 individual data points, were gathered from four different pressure drop studies: Burke and Plummer,[6] Ergun and Orning,[7] Oman and Watson,[8] and Wentz and Thodos.[9] As the data were analyzed, it was found that the Oman and Watson data were consistently higher than predicted by any model. Variation about the other data sets showed no trends (random variation). This suggested systematic variations with the Oman and Watson data rather than random data variations. Therefore, in this study Oman and Watson's data have been omitted leaving 324 data points.

The three data sets we considered included a wide range of Re, bed porosities (ε_b), and d_t/d_p ratios as shown in Table 1. Flow regimes covered by the data include: laminar to turbulent flow including the transition region, large wall effects ($d_t/d_p < 10$), and high porosity. The variety of flow regimes represented by the data allowed us to compare the combined correlation against the published correlations in their applicable ranges.

Results and Discussion

The data analysis for this study is presented in Figure 1 and Table 2. Figure 1 compares the experimental data in the ranges shown in Table 1 to the predicted values from four of the correlations considered earlier in this article. The correlations shown are the LME, Tal, MH, and TL correlations. The TL correlation fits the data very well over the whole range of Re. The next best fit as shown in the figure and as confirmed by the ARAE in Table 2 is the Tal correlation. The only deviation of the Tal correlation is for low Re

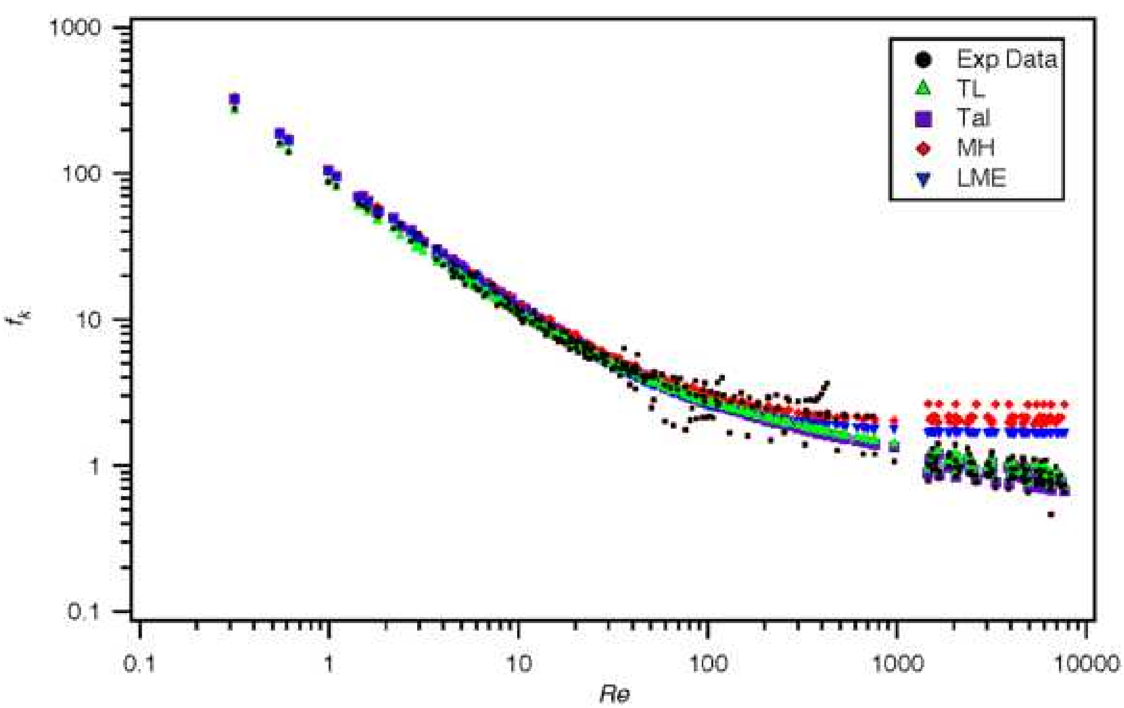

Figure 1. Friction factor as a function of Re for three published correlations (LME, Tal, and MH) and the preferred combined correlation (TL) compared to the 324 experimental data points from the literature.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

Table 2. Percent Average Relative Absolute Error (ARAE) between $\Delta P/L$ Data and the Five Previously Reported Correlations and the Two Combined Correlations

	# points	Ergun	Tal[a]	MH[b]	Liu[c]	LME[d]	Combined Correlations TMH[e]	Combined Correlations TL[f]
$Re < 2300$	252	22.7	26.7	22.5	27.4	21.4	22.8	14.7
$2300 < Re > 4000$	26	57.1	35.3	108	40.8	82.6	14.0	8.5
$Re > 4000$	46	101	15.7	157	56.3	103.6	14.3	9.1
$d_t/d_p < 10$	50	28.3	30.8	32.8	42.1	25.8	34.3	27.8
$10 < d_t/d_p < 20$	196	48.3	28.6	66.1	33.5	50.7	22.3	11.7
$d_t/d_p > 20$	78	12.3	15.6	14.2	24.1	13.9	8.7	8.4
Overall	324	36.5	25.8	48.4	32.6	38.0	20.9	13.4

[a]Tallmadge.
[b]Mehta-Hawley.
[c]Liu et al. (1-D).
[d]Liu et al. modified Ergun.
[e]Combined Tallmadge and Mehta-Hawley.
[f]Combined Tallmadge and Liu et al.

where it predicts slightly higher pressure drops than the experimental data. The biggest deviations shown in the figure are with both the MH and LME correlations in the high *Re* region. This observation is consistent with the data in Table 2 where both correlations increased in ARAE as *Re* increased. This is also consistent from a phenomenological perspective because at high *Re* both of these correlations predict a constant value of 1.75 for the f_k. This can be seen on the graph as both correlations approach a zero slope at high *Re*. Tal and TL include *Re* functionality in the turbulent term and, thus, are able to continue to track the experimental data.

Table 2 reports the ARAE of the five published correlations and the two combined correlation for three ranges of *Re* and three ranges of d_t/d_p. It also includes an overall ARAE over the full data set (324 points) for each correlation. From a phenomenological perspective, the wall-effect can be split into three regimes, bulk flow region ($d_t/d_p > 20$), transition region ($10 < d_t/d_p < 20$), and the wall-effect region ($d_t/d_p < 10$). The wall-effect terms (included in MH, LME, Liu, and TL) are unity in the bulk flow region, deviate very slightly from unity in the transition region, and deviate appreciably from unity in the wall-effect region. Thus, their contribution to f_k is slightly felt at $d_t/d_p < 20$ and makes a strong contribution at $d_t/d_p < 10$. The TL correlation yielded the lowest ARAE in all regions except for $d_t/d_p < 10$. The best predictor for this region was the LME correlation with an ARAE of 25.8%. At 27.8%, the ARAE of the TL correlation was very close to the LME correlation. It should be noted that the range of ARAE for this region for all the correlations was between 25.8 and 42.1% with none of them standing out as great predictors for this region. As the d_t/d_p ratio increased, the TL correlation was by far the best predictor of pressure drop. For example, the region on the fringe of significant wall-particle interactions, $10 < d_t/d_p < 20$, showed great disparity between the TL correlation which had an ARAE of 11.7% and the MH and LME correlations which had ARAE of 66.1 and 50.7%, respectively. It should be noted that this subset of 162 data points included all of the Wentz and Thodos data (97 data points) which were all at high *Re* that laminar-regime correlations (MH and LME) would not have been expected to predict well.

The TL correlation had the lowest ARAE of all the correlations considered for each of the three *Re* regions. As expected, the errors for the correlations developed specifically for low *Re*, that is, Ergun, MH, Liu, and LME, increased as the *Re* increased. The opposite trend was seen with the TL correlation. For the data in the turbulent region ($Re > 4000$), the ARAE of the TL correlation was an excellent 9.1% compared to Tal at 15.7%, Liu at 56.3%, and the remaining three published laminar correlations which were all over 100%.

Summary

In summary, the proposed correlation of this study obtained by combining the Tallmadge and Liu correlation predicts pressure drops for packed beds over a wide range of Reynolds numbers and tube-diameter to particle-diameter ratios more accurately than any other correlation analyzed. Its overall ARAE for the full data set (ranges in Table 1) was 13.4%. For comparison, the TMH combined correlation gave an ARAE of 20.9%. The Tallmadge correlation had the best accuracy of the literature correlations with an overall ARAE of 25.8%. The other correlations' overall ARAE ranged from 32.6% for the Liu correlation to 48.4% for the MH correlation. Combining the portions of each correlation meant to improve the applicability of the original Ergun correlation in different flow regimes has created a single correlation that is accurate over all the different flow regimes. The combined correlation can be used with confidence over $0.32 < Re < 7{,}700$, $0.33 < \varepsilon_b < 0.88$, and $8.3 < d_t/d_p < 50$ to predict pressure drop in packed beds.

Notation

d_{p_e} = effective spherical particle diameter
d_t/d_p = ratio of tube diameter to pellet diameter
d_t = tube diameter
ε_b = bed porosity
f_k = friction factor
K_{Lam} = Laminar-term coefficient
K_{Turb} = Turbulent-term coefficient
$\Delta P/L$ = pressure drop per tube length
ρ = density
n = number of values

Re = Reynolds number $= d_{p_e} u\rho/\mu$
u = superficial velocity
μ = viscosity
x'_i = predicted $\Delta P/L$
x_i = measured $\Delta P/L$

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